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LETTER

Simple method for intermodulation products counting in multicarrier systems

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Abstract

This paper presents a novel algorithm for intermodulation products counting in multicarrier systems. Intermodulation products are caused by the nonlinearity of the amplifiers' transfer characteristic. Along the transmission network, broadband amplifiers are needed to boost up the signal level and compensate the signal attenuation. When the number of carriers increases, the number of intermodulation products also increases extremely fast. In our approach, selective counting procedure by grouping intermodulation products was introduced. The presented solution shows considerable reduction in computational effort in counting the number of intermodulation products in the individual channel, especially if the number of products is very large.

Keywords: Multicarrier systems; Nonlinear distortion; Intermodulation products; CTB; CSO

1. Introduction

When several carriers are transmitted through a system with nonlinear characteristic, a large number of unwanted intermodulation products (IM) is generated [1–3]. Distortion products can include second- and third-order harmonics and intermodulation products, as well as double and triple beat products [4]. Frequency products due to second-order nonlinearity are $A \pm B$ and $2A$. Grouped together, $A \pm B$ products form composite second-order beats (CSO). Frequency components due to third-order nonlinearity at $A \pm B \pm C$, $3A$ and $2A - B$ form third-order products. Composite triple beat (CTB) distortion is formed from $A \pm B \pm C$ components. There are several algorithms for sorting and counting third-order IM products generated by a large number of carriers in multicarrier systems [5–8]. Knowledge of the number of IM products per frequency is necessary for the evaluation of the intermodulation power spectrum [4]. Knowing the total number of CTB beats in the single channel and the amplitude of a single beat component, we can calculate the level of the disturbance for any channel. In multicarrier systems, with equal channel spacing, various IM products generated in the amplifiers fall at the same frequency, forming beats, which results in very high levels of interference at certain frequencies. The problem is in a very large number of intermodulation products that increases extremely quick with increasing the number of carriers in the system. CTB products usually present the most significant problem in multicarrier systems. In a system with 4 carriers 16 CTB components are produced, for 5 carriers 40 CTB components, for 8 carriers 224 components, and in a 10-channel system there can be 480 CTB components. Therefore, direct counting of CTB products in a single channel is complex and time consuming.

In the method presented in [5] the frequency distribution and carrier configuration of IM products are described by

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polynoms, and the frequency distribution of the third-order IM products are obtained with the aid of a FFT algorithm and a numerical method for solving a set of linear equations. Although this technique significantly reduces the computational effort compared to the direct counting method (DCM) in a system with a large number of carriers, the algorithm is not simple. It requires a large number of multiplications and additions. In [6] the authors present three selective counting procedures: sorting by position, sorting by position and group, sorting by position, group and kind; based on a discrete third-order Voltera model. This algorithm provides the exact counting of third order products, but all the information about the carriers should be provided (number of carriers, type of carriers, frequency positions, magnitude of each carrier). In [9] the authors present an extension to the existing method for fast counting third order intermodulation products and propose a compact expression for fifth order products. A modification of the counting method from [9] which includes even order products is presented in [10]. Both [9,10] incorporate model of a passband memoryless nonlinear device as the cause of IM products. Carrier configuration is represented by means of indicator polynomials which result in complex relationships between carrier positions and number of products.

Our paper presents a simple method for counting the number of CTB intermodulation products in multicarrier systems. Our new method significantly reduces the computational complexity in comparison to the direct counting of CTB products. Development of our algorithm starts with the graphical representation of CTB distribution for small number of carriers, which was used for derivation of the mathematical expression which is valid for any number of carriers in the system. Due to its computational simplicity, it is very practical, efficient and useful, considering only CTB products with the highest impact on the third-order distortion.

The paper is organized as follows: Section 2 presents a new method for counting the number of CTB products, Section 3 considers experimental results and evaluation of a new method, and conclusion of the work is provided in Section 4.

2. New method for counting the number of beats

Analysis of amplitudes of different nonlinear distortion products shows that CSO and CTB components have different levels [3]. CSO products do not contribute considerably to the distortion level, as the second-order nonlinearity is suppressed in push-pull amplifier configurations. Therefore, CSO products are not considered in this article. In cable television (CATV) systems with several cascaded amplifiers, CTB products usually present the most significant problem, as they have highest amplitudes comparing to other IM products [4]. CTB impairment in the TV picture appears as horizontal lines in the whole picture randomly moving vertically. The visual effect of CTB is very annoying, and the threshold of perceptibility is 57 dB below the carrier level [11]. Analysing relative amplitudes of third order IM components, there are different levels of single components [1,3,12]. The strongest and most significant third order products are formed as a combination of three input frequencies $A \pm B \pm C$, that fall on carrier frequencies. Third-order harmonics of $3A$ are usually 15.6 dB weaker, $2A - B$ products are 9.5 dB weaker than $A \pm B \pm C$ products, and they do not contribute considerable to the amplitude of intermodulation products.

In our approach the major assumption used for counting the number of CTB products per channel is that all carriers are equally spaced in frequency, what is usual in multicarrier systems. CATV systems in America have channel spacing of 6 MHz, and in Europe 7 MHz in VHF band, and 8 MHz in UHF band. The analysis considers a real 27-channels CATV system in the VHF band from 111 to 300 MHz (channels from S2–S10, CH5–CH12, and S11–S20) with 7 MHz channel spacing.

To derive the formula for counting the number of beats per frequency, graphical representation for a small number of carriers was used. This representation was basis for analysing the frequency distribution of different third-order intermodulation products. The beats that fall within the frequency band of a CATV system were calculated by grouping the CTB products in 3 groups. These groups were $A+B-C$, $A-B+C$ and $-A+B+C$. If we presume that $A<B<C$, most CTB products have positive frequencies. Other combinations fall outside the frequency band of our interest. In our analyses the first channel was the channel S2, with the carrier frequency at 112.25 MHz. Fig. 1 shows the distribution of CTB products for these three groups in the system with 5, 6, 7 and 8 channels, obtained by direct counting method. Analysing the curves of the distribution of CTB products, we can see that for the same number of channels in a system, there is a different distribution of CTB products of these three groups. Within each group we can see similar shape of the curve for different number of carriers. This gave us an idea to find mathematical equations which describe the frequency distribution for these three groups of CTB products, valid for any number of channels.

We will define the terms for the calculation as:

- Z — total number of frequencies at which CTB products are distributed,
- x — single beat frequency,
- $f_{\min}$ — lowest frequency component with $x = 1$,
- $f_{\max}$ — highest frequency component with $x = Z$,
- N — total number of channels in the system,
- M — actual channel of interest,
- f_l — frequency of the lowest channel ($M = 1$),
- f_h — frequency of the highest channel ($M = N$),
- f_r — frequency spacing of the channels

(all frequencies are in MHz).

Analysing the curves in Fig. 1(a) for the group $A + B - C$, we defined terms for the lowest and highest frequency component of these products ($f_{\min}$, $f_{\max}$), as well as the total

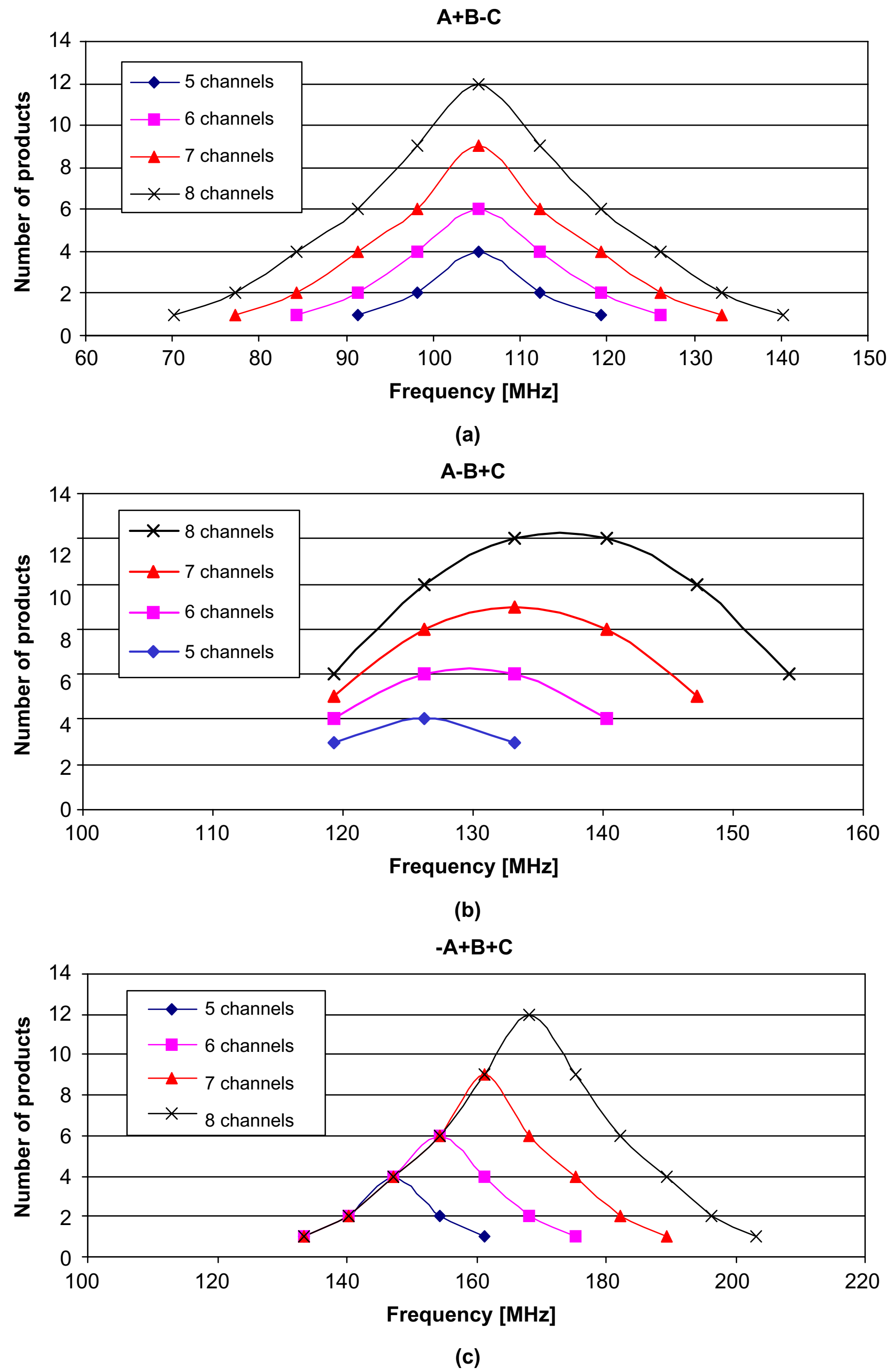

Fig. 1. Distribution of CTB products for: (a) $A + B - C$, (b) $A - B + C$, (c) $-A + B + C$.

number of frequencies at which CTB products are distributed (Z), valid for any number of channels:

$$Z = N + (N - 5), \quad f_{\min} = 2f_l - f_h + f_r,$$
$$f_{\max} = f_h - 3f_r.$$

Frequency with the maximum number of products is the frequency f_M which is defined as

$$f_M = (f_{\min} + f_{\max})/2 = f_l - f_r.$$

The lowest frequency occupied with CTB products of $A + B - C$ group ($f_{\min}$) with $x = 1$ is lower than the frequency of the first channel in the system (S2 at 112.25 MHz). The frequency of the lowest single beat of this group depends on the number of channels in the system. The lowest channel degraded with these products is the first channel in the system where $M = 1$. At the first channel S2 we have $x = (Z + 3)/2$. The highest channel occupied with this group of products with $M = N - 4$, is at the frequency $f_{\max}$ where we have

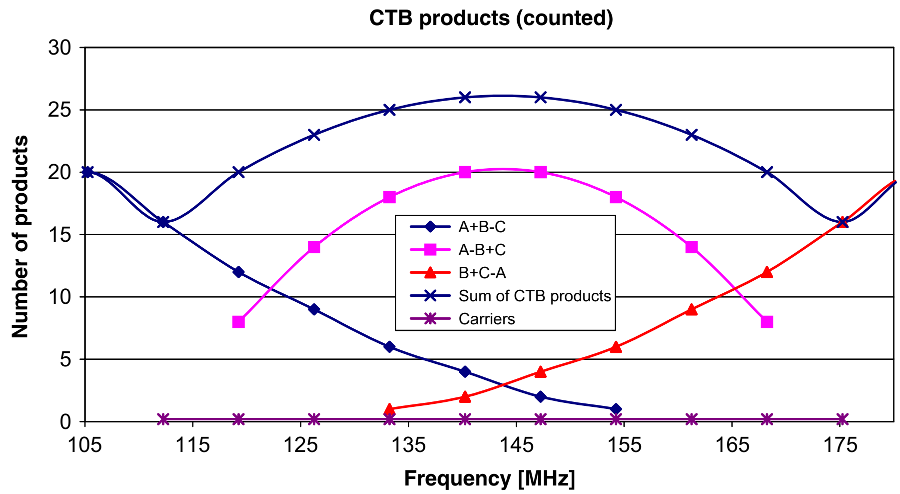

Fig. 2. Actual distribution of CTB products in a 10-channel system.

$x = Z$. Above this channel CTB products due to $A + B - C$ group do not exist. For example in the 5-channel system we have $N = 5$; $Z = 5$; $f_{\min} = 91.25$ MHz; $f_{\max} = 119.25$ MHz; $f_M = 105.25$ MHz.

In our approach we wanted to express the number of CTB components within the frequency band used for the transmission, for every channel (M) in dependence on the number of channels in the system (N).

For the group $A + B - C$ the shape of the curves, Fig. 1(a), for the number of products B_X in every channel of interest can be expressed with the approximation:

$$B_{X(A+B-C)} = [(N - M)^2 - 2(N - M)]/4. \quad (1)$$

Following the similar procedure as for the group $A + B - C$, for the group $A - B + C$ we achieve:

$$Z = N - 2, \quad f_{\min} = f_l + f_r, \quad f_{\max} = f_h - f_r.$$

Frequency with the maximum number of products is the frequency f_M which is defined as

$$f_M = (f_{\min} + f_{\max})/2 = (f_l + f_h)/2.$$

The shape of the curves from Fig. 1(b) for the number of products B_X changing from $f_{\min}$ with $x = 1$, to $f_{\max}$ with $x = Z$, can be expressed by the equation $B_X = x(N - (x + 1))$. If we want to use this formula for the actual channel, we replace x with $M - 1$, as for this group is $x = M - 1$. The number of products B_X at single frequency in every channel of interest can be expressed as

$$B_{X(A-B+C)} = (M - 1)(N - M). \quad (2)$$

For the group $-A + B + C$ we get:

$$Z = N + (N - 5), \quad f_{\min} = f_l + 3f_r,$$

$$f_{\max} = f_h + (N - 2)f_r.$$

Frequency with the maximum number of products is the frequency f_M which is defined as

$$f_M = (f_{\min} + f_{\max})/2 = f_h + f_r.$$

From the Fig. 1(c) we noticed that the curves overlap within the frequency band used for the transmission, independent of the number of channels (N). The shape of the curves for the number of products B_X changing from $f_{\min}$ ($x = 1$, $M = 4$), to the highest channel ($M = N, x = (Z - 1)/2$) can be expressed by the approximation:

$$B_{X(B+C-A)} = [(M - 1)^2 - 2(M - 1)]/4$$
$$= (M^2 - 4M + 3)/4. \quad (3)$$

All analysed groups have the same highest number of products $B_{X_{\max}}$. The equation that approximates the maximum number of products for each of these three groups can be expressed as

$$B_{X_{\max}} = (N^2 - 2N)/4. \quad (4)$$

Summing up Eqs. (1)–(3), we get the total number of CTB products in each channel of interest:

$$B_{X_{\text{sum}}} = \frac{(M + N)^2 - 3M^2 + 2M - 6N + 3}{4}. \quad (5)$$

Eq. (5) is valid for the systems with any number of channels with equally spaced carrier frequencies. It is very simple and remarkably accurate when compared to direct counting of CTB products in systems with large number of carriers, as we will prove in the next section.

3. Experimental results and evaluation

We compared the actual distribution of CTB products in the system with 10 channels, Fig. 2, with the calculated curves of the distribution of CTB products according to

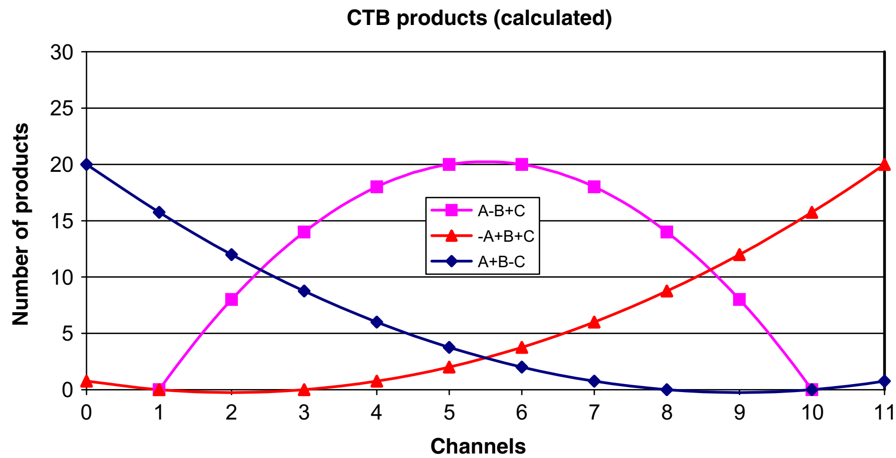

Fig. 3. Calculated distribution of CTB products in a 10-channel system.

Table 1. Number of intermodulation products derived by DCM and our new method

Channel	Frequency band (MHz)	Frequency (MHz)	Number of CTB products			Real number of CTB products (DCM)	Number of CTB products using our new method
			$A+B+C$	$A+B-C$	$A-B+C$		
S2	111–118	112.25	0	156	0	156	156
S3	118–125	119.25	0	144	25	169	168.5
S4	125–132	126.25	0	132	48	180	180
S5	132–139	133.25	1	121	69	191	190.5
S6	139–146	140.25	2	110	88	200	200
S7	146–153	147.25	4	100	105	209	208.5
S8	153–160	154.25	6	90	120	216	216
S9	160–167	161.25	9	81	133	223	222.5
S10	167–174	168.25	12	72	144	228	228
K5	174–181	175.25	16	64	153	233	232.5
K6	181–188	182.25	20	56	160	236	236
K7	188–195	189.25	25	49	165	239	238.5
K8	195–202	196.25	30	42	168	240	240
K9	202–209	203.25	36	36	169	241	240.5
K10	209–216	210.25	42	30	168	240	240
K11	216–223	217.25	49	25	165	239	238.5
K12	223–230	224.25	56	20	160	236	236
S11	230–237	231.25	64	16	153	233	232.5
S12	237–244	238.25	72	12	144	228	228
S13	244–251	245.25	81	9	133	223	222.5
S14	251–258	252.25	90	6	120	216	216
S15	258–265	259.25	100	4	105	209	208.5
S16	265–272	266.25	110	2	88	200	200
S17	272–279	273.25	121	1	69	191	190.5
S18	279–286	280.25	132	0	48	180	180
S19	286–293	287.25	144	0	25	169	168.5
S20	293–300	294.25	156	0	0	156	156

Eqs. (1)–(3), Fig. 3. If we compare Figs. 2 and 3, we can notice very good conformity of the curves.

The parts of the curves for $A+B-C$ and $-A+B+C$ products outside of the frequency band used for the transmission (the band below the first transmitted channel and above the highest transmitted channel), are not defined with formulas for counting the number of CTB products per channel.

To demonstrate the features of our new method for counting CTB products we analysed the distribution of CTB products in the 27-channel system. Table 1 shows comparison

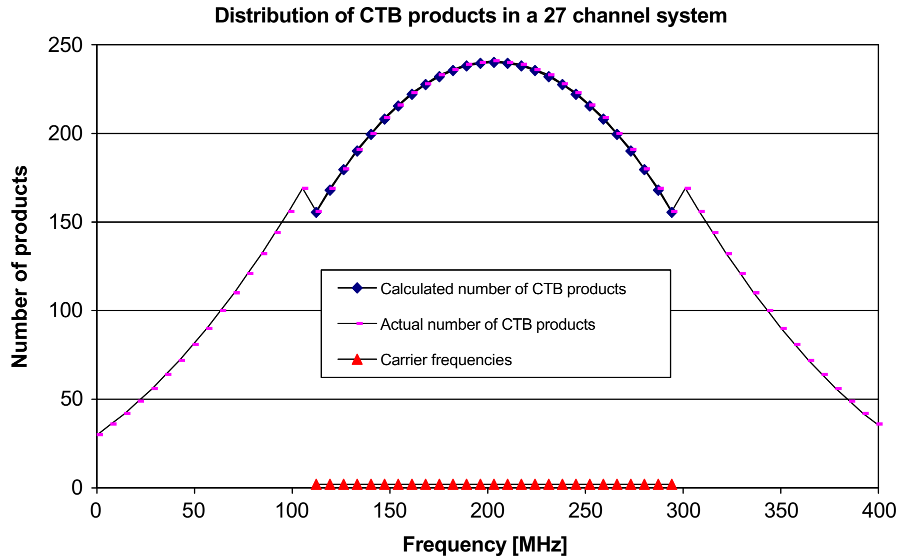

Fig. 4. Comparison of actual number of CTB products with the calculation.

between the number of intermodulation products as a result of DCM and the number of intermodulation products using our new method. As it can be seen from Fig. 4, there is practically no difference between the calculated and the actual number of CTB products. The total difference between the actual number of products and the calculation is less than 0.5, which can be neglected. As the number of CTB products is always an integer, the results achieved using Eq. (5) can be rounded towards the next higher integer which would make them exact.

For the illustration of the advantages of our CTB counting method we have to take into account that in a 5-channel system 40 CTB components are generated, in a 10-channel system 480 components, and in a 27-channel system even 11,700 CTB components, which have to be calculated and sorted out in a DCM method. Our counting method requires only one calculation per channel to get the exact number of beats for every channel.

4. Conclusion

In this paper, we presented a new method for counting of CTB distortion components in multicarrier systems. We derived equations for selective counting of CTB intermodulation products that can be used in any system, if carrier frequencies have the same channel spacing. We proved that our formula shows remarkable correspondence with the actual distribution of CTB products derived by direct counting method. It is very practical, efficient and useful when many channels are transmitted through the system with a nonlinear transfer characteristic. Due to its computational simplicity our new method can significantly reduce the computational effort compared to the direct counting of CTB products, especially if the number of channels in the system is large.

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